Beispiel 4:

$$Funktion: \quad f(x,y) = x^k y^m - 4x^t + 5y^2$$

$$Partielle\ Ableitungen: \quad f_x(x,y) = ??? \qquad\qquad f_y(x,y) = ???$$

$$f(x,y) = x^k y^m - 4x^t + 5y^2$$

$$f_x(x,y) = kx^{k-1}y^m - 4tx^{t-1} \qquad\qquad f_y(x,y) = mx^k y^{m-1} + 10y$$

Beispiel 5:

$$Funktion: \quad f(x,y) = x^\alpha y^\beta - 4x^{5k} + \frac{1}{2}y^{4k}$$

$$Partielle\ Ableitungen: \quad f_x(x,y) = ??? \qquad\qquad f_y(x,y) = ???$$

$$f(x,y) = x^\alpha y^\beta - 4x^{5k} + \frac{1}{2}y^{4k}$$

$$f_x(x,y) = \alpha x^{\alpha-1}y^\beta - 20kx^{5k-1} \qquad\qquad f_y(x,y) = \beta x^\alpha y^{\beta-1} + 2ky^{4k-1}$$

a) $f(x,y) = 2x^3 - 24x - 18y + 3y^2$

$$f(x/y) = 2x^3 - 24x - 18y + 3y^2$$

$$\left.\begin{array}{l} f_x(x/y) = 6x^2 - 24 = 0 \rightarrow x_1 = 2 \quad und \quad x_2 = -2 \\ f_y(x/y) = -18 + 6y = 0 \rightarrow y = 3 \end{array}\right\} \begin{array}{l} S_1(2 \quad 3 \quad f_1) \\ S_2(-2 \quad 3 \quad f_2) \end{array}$$

$Hesse - Matrix:$

$$H(f) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \rightarrow H(f) = \begin{pmatrix} 12x & 0 \\ 0 & 6 \end{pmatrix}$$

$Auswertung:$

$$H_{S_1}(f) = \begin{pmatrix} 12 \cdot 2 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 24 & 0 \\ 0 & 6 \end{pmatrix} \xrightarrow{Auswertung} \left.\begin{array}{l} f_{xx} = 24 > 0 \\ \det\begin{pmatrix} 24 & 0 \\ 0 & 6 \end{pmatrix} = 144 > 0 \end{array}\right\} \begin{array}{l} positiv\ definit \\ \rightarrow Minimum \end{array}$$

$$f(2/3) = 16 - 48 - 54 + 27 = -59$$

$$H_{S_2}(f) = \begin{pmatrix} 12 \cdot (-2) & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} -24 & 0 \\ 0 & 6 \end{pmatrix} \xrightarrow{Auswertung} \left.\begin{array}{l} f_{xx} = -24 < 0 \\ \det\begin{pmatrix} -24 & 0 \\ 0 & 6 \end{pmatrix} = -144 < 0 \end{array}\right\} \begin{array}{l} indefinit \\ \rightarrow SP \end{array}$$

d) $$f(x,y) = \frac{1}{3}x^3 - x^2 + y^3 - 12y$$

$$f(x/y) = \frac{1}{3}x^3 - x^2 + y^3 - 12y$$

$$\left.\begin{array}{l} f_x(x/y) = x^2 - 2x = 0 \rightarrow x_1 = 0 \quad und \quad x_2 = 2 \\ f_y(x/y) = 3y^2 - 12 = 0 \rightarrow y_1 = -2 \quad und \quad y_2 = 2 \end{array}\right\} \begin{array}{ll} S_1(0 \quad -2 \quad f_1) & S_3(2 \quad -2 \quad f_3) \\ S_2(0 \quad 2 \quad f_2) & S_4(2 \quad 2 \quad f_4) \end{array}$$

$Hesse-Matrix:$

$$H(f) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \rightarrow H(f) = \begin{pmatrix} 2x-2 & 0 \\ 0 & 6y \end{pmatrix}$$

$Auswertung:$

$$H_{S_1}(f) = \begin{pmatrix} -2 & 0 \\ 0 & -12 \end{pmatrix} \xrightarrow{Auswertung} \left.\begin{array}{l} f_{xx} = -2 < 0 \\ \det\begin{pmatrix} -2 & 0 \\ 0 & -12 \end{pmatrix} = 24 > 0 \end{array}\right\} \begin{array}{l} negativ\ definit \\ \rightarrow Maximum \end{array}$$

$$H_{S_2}(f) = \begin{pmatrix} -2 & 0 \\ 0 & 12 \end{pmatrix} \xrightarrow{Auswertung} \left.\begin{array}{l} f_{xx} = -2 < 0 \\ \det\begin{pmatrix} -2 & 0 \\ 0 & 12 \end{pmatrix} = -24 < 0 \end{array}\right\} \begin{array}{l} indefinit \\ \rightarrow SP \end{array}$$

$$H_{S_3}(f) = \begin{pmatrix} 2 & 0 \\ 0 & -12 \end{pmatrix} \xrightarrow{Auswertung} \left.\begin{array}{l} f_{xx} = 2 > 0 \\ \det\begin{pmatrix} 2 & 0 \\ 0 & -12 \end{pmatrix} = -24 < 0 \end{array}\right\} \begin{array}{l} indefinit \\ \rightarrow SP \end{array}$$

$$H_{S_4}(f) = \begin{pmatrix} 2 & 0 \\ 0 & 12 \end{pmatrix} \xrightarrow{Auswertung} \left.\begin{array}{l} f_{xx} = 2 > 0 \\ \det\begin{pmatrix} 2 & 0 \\ 0 & 12 \end{pmatrix} = 24 > 0 \end{array}\right\} \begin{array}{l} positiv\ definit \\ \rightarrow Minimum \end{array}$$

$$f\left(x,y\right)=\frac{1}{2}x^{2}+2xy+y^{2}+4x+2y+3$$

$$f\left(x/y\right)=\frac{1}{2}x^{2}+2xy+y^{2}+4x+2y+3$$

$$\left.\begin{array}{l} f_x\left(x/y\right)=x+2y+4 \quad = \quad 0 \\ f_y\left(x/y\right)=2x+2y+2 \quad = \quad 0 \end{array}\right\}\xrightarrow{f_y-f_x} x-2=0 \quad \rightarrow \quad x=2$$

$$\xrightarrow{in\ f_x} 2+2y+4=0 \quad \rightarrow \quad y=-3 \quad \rightarrow \quad S\left(2 \quad -3 \quad f\right)$$

$$H\left(f\right)=\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \xrightarrow{Auswertung} indefinit \quad \rightarrow \quad SP$$

3 Variablen

$$f(x,y,z) \quad = \quad 490+2x+2y-\frac{9}{2}x^2-y^2-\frac{1}{2}z^2+3xy+2xz$$

$$f(x/y/z)=490+2x+2y-\frac{9}{2}x^2-y^2-\frac{1}{2}z^2+3xy+2xz$$

$$\left.\begin{array}{l} f_x(x/y/z)=2-9x+3y+2z \quad = \quad 0 \\ f_y(x/y/z)=2-2y+3x \quad = \quad 0 \\ f_z(x/y/z)=-z+2x \quad = \quad 0 \end{array}\right\} \xrightarrow{\text{einsetzen}} \begin{array}{c} 2-9x+3\left(1+\frac{3}{2}x\right)+2\cdot 2x \quad = \quad 0 \\ y \quad = \quad 1+\frac{3}{2}x \\ z \quad = \quad 2x \end{array}$$

$$5-9x+\frac{9}{2}x+4x \quad = \quad 0 \quad \rightarrow \quad x=10 \quad \rightarrow \quad y=16 \quad \rightarrow \quad z=20$$

$$S(10 \mid 16 \mid 20 \mid f)$$

$$H(f)=\begin{pmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{pmatrix} \quad \rightarrow \quad H(f)=\begin{pmatrix} -9 & 3 & 2 \\ 3 & -2 & 0 \\ 2 & 0 & -1 \end{pmatrix}$$

Auswertung :

$$(1) \quad f_{xx}=-9<0$$

$$(2) \quad \det\begin{pmatrix} -9 & 3 \\ 3 & -2 \end{pmatrix}=18-9=9>0$$

$$(3) \quad \det\begin{pmatrix} -9 & 3 & 2 \\ 3 & -2 & 0 \\ 2 & 0 & -1 \end{pmatrix}=-1<0$$

$$\rightarrow \quad \textit{negativ definit} \quad \rightarrow \quad \textit{Minimum}$$