e-Funkton der Gauß-Normalverteilung

$$\varphi_{\mu,\sigma}(x)=\frac{1}{\sigma\cdot\sqrt{2\pi}}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard-Normalverteilung:

$$\varphi_{0,1}(x)=\frac{1}{1\cdot\sqrt{2\pi}}\cdot e^{-\frac{(x-0)^2}{2\cdot 1^2}}=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}$$

K-Disk:

Nullstellen?

$$\varphi_{0,1}(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}=0 \xrightarrow{\cdot\sqrt{2\pi}} e^{-\frac{1}{2}x^2}=0 \xrightarrow{\log arithmieren}$$

$$\ln\left(e^{-\frac{1}{2}x^2}\right)=\ln(0) \quad Problem: \ln(0)\ nicht\ definiert \quad \rightarrow \quad keine\ Nullstelle(n)$$

Sy? X = 0

$$\varphi_{0,1}(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2} \xrightarrow{x=0} \varphi_{0,1}(0)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\cdot 0^2}=\frac{1}{\sqrt{2\pi}}\cdot e^0=\frac{1}{\sqrt{2\pi}}\approx 0,3989\approx 0,4$$

$$S_y(0\mid 0,4)$$

Extrema???

$$\varphi_{0,1}(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}$$

$$1.\ Ableitung\ (=Steigung):$$

$$\frac{d\varphi}{dx}\varphi_{0,1}(x)=\left[\varphi_{0,1}(x)\right]^I \overset{Kettenregel}{=} \frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}\cdot(-x)=-\frac{x}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}=0$$

$$\xrightarrow[Nullprodukt]{Satz\ vom} -\frac{x}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}=0 \xrightarrow{e^{-\frac{1}{2}x^2}\neq 0} -\frac{x}{\sqrt{2\pi}}=0 \rightarrow x=0$$

$$\rightarrow \varphi_{0,1}(0)\approx 0,4 \rightarrow Extremum: (0\mid 0,4)???$$

$$2.\ Ableitung\,(=Krümmung):$$

$$\left[\varphi_{0,1}(x)\right]^{II} \underset{Produktregel}{\overset{Kettenregel}{=}} \left(-\frac{1}{\sqrt{2\pi}}\right)\cdot e^{-\frac{1}{2}x^2} + \left(-\frac{x}{\sqrt{2\pi}}\right)\cdot e^{-\frac{1}{2}x^2}\cdot(-x)$$

$$\left[\varphi_{0,1}(x)\right]^{II} = e^{-\frac{1}{2}x^2}\cdot\left[\left(-\frac{1}{\sqrt{2\pi}}\right)+\left(\frac{x^2}{\sqrt{2\pi}}\right)\right] = e^{-\frac{1}{2}x^2}\cdot\left(\frac{x^2-1}{\sqrt{2\pi}}\right)$$

Prüfung der Extremwertstelle x = 0:

$$\left[\varphi_{0,1}(0)\right]^{II} = e^{0}\cdot\left(\frac{0^2-1}{\sqrt{2\pi}}\right) = 1\cdot\left(\frac{-1}{\sqrt{2\pi}}\right)<0 \Rightarrow Max:(0\mid 0,4)$$

Wendepunkte???

$$\varphi_{0,1}(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}$$

$$2.\ Ableitung\,(=Krümmung):$$

$$\left[\varphi_{0,1}(x)\right]^{II} = e^{-\frac{1}{2}x^2}\cdot\left(\frac{x^2-1}{\sqrt{2\pi}}\right)=0$$

$$\xrightarrow[Nullprodukt]{Satz\ vom} e^{-\frac{1}{2}x^2}\cdot\left(\frac{x^2-1}{\sqrt{2\pi}}\right)=0 \xrightarrow{e^{-\frac{1}{2}x^2}\neq 0} \frac{x^2-1}{\sqrt{2\pi}}=0 \rightarrow |x|=1$$

$$Funktionswert:$$

$$\varphi_{0,1}(\pm 1)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}}\approx 0,24\,[Achsensymmetrie]$$

$$3.\ Ableitung\,(=Krümmung):$$

$$\left[\varphi_{0,1}(x)\right]^{III} \underset{Produktregel}{\overset{Kettenregel}{=}} e^{-\frac{1}{2}x^2}\cdot(-x)\cdot\left(\frac{x^2-1}{\sqrt{2\pi}}\right)+e^{-\frac{1}{2}x^2}\cdot\left(\frac{2x}{\sqrt{2\pi}}\right)$$

$$\left[\varphi_{0,1}(x)\right]^{III} = e^{-\frac{1}{2}x^2}\cdot\left[\left(\frac{-x^3+x}{\sqrt{2\pi}}\right)+\left(\frac{2x}{\sqrt{2\pi}}\right)\right] = e^{-\frac{1}{2}x^2}\cdot\left(\frac{-x^3+3x}{\sqrt{2\pi}}\right)$$

Prüfung der Wendestelle x = 1:

$$\left[\varphi_{0,1}(1)\right]^{II} = e^{-\frac{1}{2}}\cdot\frac{2}{\sqrt{2\pi}} = e^{-\frac{1}{2}}\cdot\sqrt{\frac{2}{\pi}}\neq 0 \Rightarrow W_{1/2}(\pm 1\mid 0,24)$$

Tangenten in den Wendepunkten:

Steigung in x = 1:

$$Funktionswert\ in\ W\left(\pm 1 \mid 0{,}24\right): \quad \varphi_{0,1}\left(\pm 1\right) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\cdot(\pm 1)^2} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}}$$

$$Steigung\ m\ in\ x=1: \quad \left[\varphi_{0,1}\left(1\right)\right]^I = -\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}}$$

$$y-Achsenabschnitt: \quad t_1\left(x\right) = mx + b$$

$$\rightarrow \quad \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} = -\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot 1 + b \quad \xrightarrow{+\frac{1}{\sqrt{2\pi}}} \quad b = \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}}$$

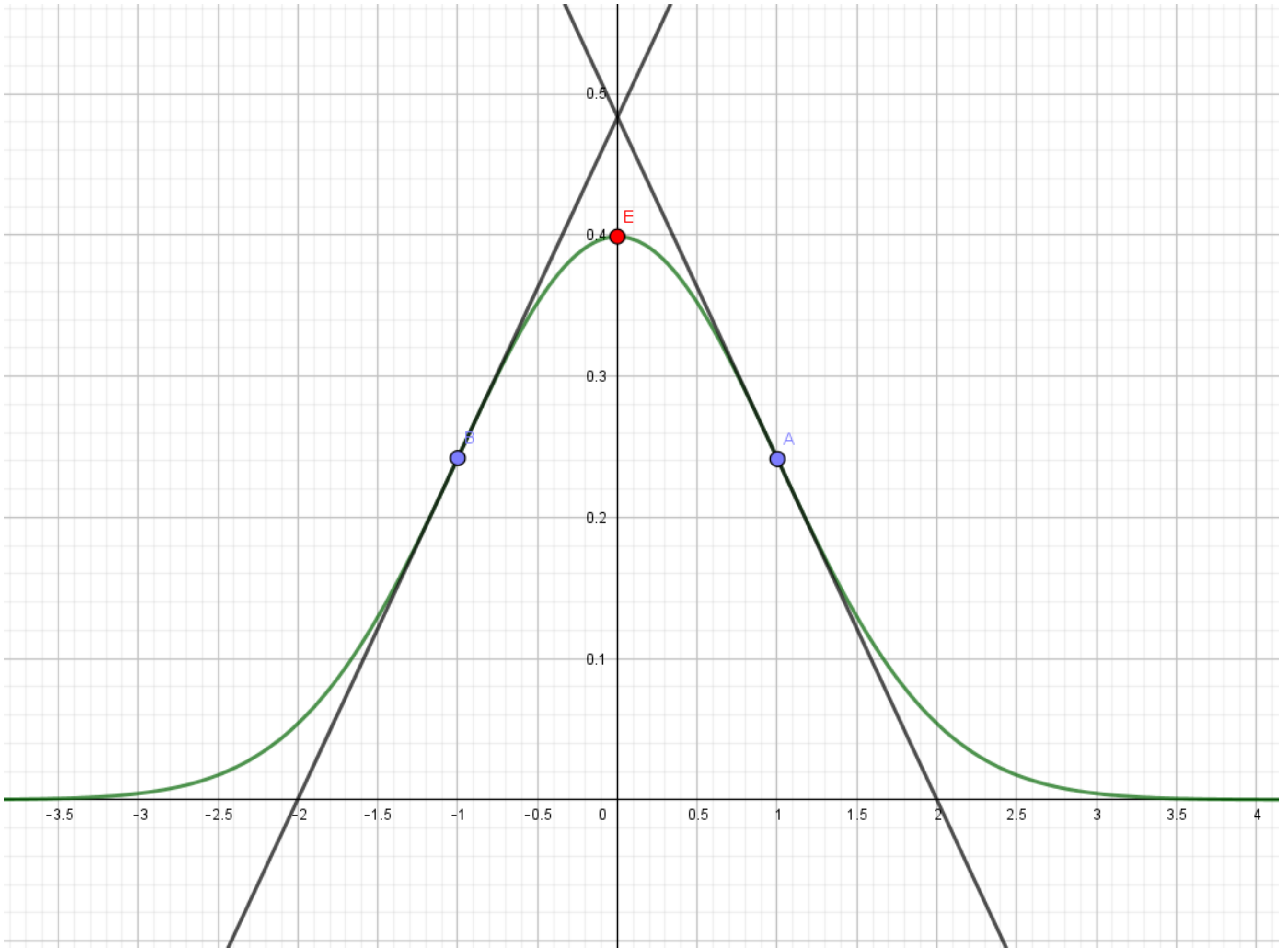
$$\xrightarrow{Tangente} \quad t_1\left(x\right) = -\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot x + \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot \left(2 - x\right)$$

$$Steigung\ m\ in\ x=-1: \quad \left[\varphi_{0,1}\left(-1\right)\right]^I = -\frac{\left(-1\right)}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}}$$

$$y-Achsenabschnitt: \quad t_2\left(x\right) = mx + b$$

$$\rightarrow \quad \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot \left(-1\right) + b \quad \xrightarrow{+\frac{1}{\sqrt{2\pi}}} \quad b = \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}}$$

$$\xrightarrow{Tangente} \quad t_2\left(x\right) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot x + \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot \left(2 + x\right)$$



0.5
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