Lösungen Arbeitsblatt: Σ & ! & Π

$a) \quad \sum_{i=1}^{6} \frac{i}{i+2}$

$b) \quad \sum_{i=1}^{6} (-1)^{i+1} \frac{i}{i+2}$

$c) \quad \sum_{i=0}^{3} \frac{1}{10^i} \quad = \quad \sum_{i=0}^{3} 10^{-i}$

$d) \quad \sum_{i=1}^{6} \frac{1}{i^2}$

$e) \quad \sum_{i=1}^{6} (-1)^{i+1} \frac{1}{i^2}$

$f) \quad \sum_{i=2}^{6} (-1)^i \cdot i$

$g) \quad \sum_{i=1}^{5} \frac{1}{2^i} \quad = \quad \sum_{i=1}^{5} 2^{-i}$

$h) \quad \sum_{i=1}^{5} 3^i$

Aufgabe 2:

$a) \quad \sum_{i=1}^{5} (k-1) \cdot k = 40$

$b) \quad \sum_{i=1}^{5} 10^{i-1} = 11.111$

$c) \quad \sum_{i=0}^{3} (-1)^i \frac{i}{1+i} = -\frac{7}{12}$

$d) \quad \sum_{t=0}^{99} (2t+1) - \sum_{t=1}^{100} (t+1) = 4.850$

$$e) \quad \sum_{t=1}^{2} \sum_{i=1}^{4} (t+2i) \quad = \quad (1+2)+(1+4)+(1+6)+(1+8)$$

$$+(2+2)+(2+4)+(2+6)+(2+8)$$

$$\sum_{t=1}^{2} \sum_{i=1}^{4} (t+2i) \quad = \quad 52$$

$$f) \quad \sum_{i=1}^{3} \frac{1+i}{i!} \quad = \quad \frac{2}{1}+\frac{3}{2}+\frac{4}{6} \quad = \quad \frac{2}{1}+\frac{3}{2}+\frac{2}{3} \quad = \quad \frac{12}{6}+\frac{9}{6}+\frac{4}{6} \quad = \quad \frac{25}{6}$$

Aufgabe 3:

$$a)\quad \frac{3!}{5!}=\frac{1}{20}=0,05 \qquad b)\quad \frac{7!}{6!}=7$$

$$c)\quad \frac{(n+1)!}{n!}=n+1 \qquad d)\quad \frac{n!}{n-1}=(n-2)!\cdot n$$

$$e)\quad \frac{n!}{(n-1)!}=n \qquad f)\quad \frac{(n+1)!}{(n-1)!}=n\cdot(n+1)$$

$$g)\quad (n+1)\cdot n!=(n+1)!$$

Aufgabe 4:

$$a)\quad \binom{7}{4}=35 \qquad b)\quad \binom{30}{1}=30$$

$$c)\quad \binom{12}{10}=66 \qquad d)\quad \binom{12}{2}=66$$

$$e)\quad \binom{5}{3}+\binom{5}{4}=\binom{6}{4}=15$$

$$f)\quad \binom{3}{4}=n.d. \qquad g)\quad \binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k}$$

Aufgabe 5:

$$a)\quad Beh.:\quad \binom{n}{0}=\binom{n}{n}=1$$

Beweis

$$(i)\quad \binom{n}{0}=\frac{n!}{0!\cdot(n-0)!}=\frac{n!}{(n-0)!\cdot 0!}=\frac{n!}{n!\cdot(n-n)!}=\binom{n}{n}$$

$$(ii)\quad \binom{n}{0}=\frac{n!}{0!\cdot(n-0)!}=\frac{n!}{1\cdot n!}=1$$

$$b)\quad Beh.:\quad \binom{n}{k}=\binom{n}{n-k}$$

Beweis

$$\binom{n}{k}=\frac{n!}{k!\cdot(n-k)!}=\frac{n!}{(n-k)!\cdot k!}$$

$$=\frac{n!}{(n-k)!\cdot\left[n-(n-k)\right]!}=\binom{n}{n-k}$$

$$c)\quad Beh.:\quad \binom{n}{k}+\binom{n}{k+1}=\binom{n+1}{k+1}$$

Beweis

$$\frac{n!}{k!\cdot(n-k)!}+\frac{n!}{(k+1)!\cdot(n-k-1)!}=\frac{n!\cdot(k+1)+n!\cdot(n-k)}{(k+1)!\cdot(n-k)!}$$

$$=\frac{n!\cdot(n+1)}{(k+1)!\cdot(n-k)!}=\frac{(n+1)!}{(k+1)!\cdot(n+k+1-1)!}$$

$$=\frac{(n+1)!}{(k+1)!\cdot\left[n+1-(k-1)\right]!}=\binom{n+1}{k+1}$$

Aufgabe 6:

$$a)\quad (x+2)^5 = \binom{5}{0}x^5 + \binom{5}{1}2x^4 + \binom{5}{2}4x^3 + \binom{5}{3}8x^2 + \binom{5}{4}16x^1 + \binom{5}{5}32$$

$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

$$b)\quad \left(\frac{1}{2}x - 4y\right)^4 = \binom{4}{0}\frac{1}{16}x^4 + \binom{4}{1}\frac{1}{8}x^3(-4)y + \binom{4}{2}\frac{1}{4}x^2 16y^2 + \binom{4}{3}\frac{1}{2}x^1(-64)y^3 + 256y^4$$

$$= \frac{1}{16}x^4 - 2x^3y + 24x^2y^2 - 128xy^3 + 256y^4$$