

Lösungen Arbeitsblatt: Σ & ! & Binomischer Lehrsatz

$$a) \sum_{i=1}^6 \frac{i}{i+2}$$

$$b) \sum_{i=1}^6 (-1)^{i+1} \frac{i}{i+2}$$

$$c) \sum_{i=0}^3 \frac{1}{10^i} = \sum_{i=0}^3 10^{-i}$$

$$d) \sum_{i=1}^6 \frac{1}{i^2}$$

$$e) \sum_{i=1}^6 (-1)^{i+1} \frac{1}{i^2}$$

$$f) \sum_{i=2}^6 (-1)^i \cdot i$$

$$g) \sum_{i=1}^5 \frac{1}{2^i} = \sum_{i=1}^5 2^{-i}$$

$$h) \sum_{i=1}^5 3^i$$

Aufgabe 2:

$$a) \sum_{i=1}^5 (k-1) \cdot k = 40$$

$$b) \sum_{i=1}^5 10^{i-1} = 11.111$$

$$c) \sum_{i=0}^3 (-1)^i \frac{i}{1+i} = -\frac{7}{12}$$

$$d) \sum_{t=0}^{99} (2t+1) - \sum_{t=1}^{100} (t+1) = 4.850$$

$$e) \sum_{t=1}^2 \sum_{i=1}^4 (t+2i) = (1+2)+(1+4)+(1+6)+(1+8) \\ + (2+2)+(2+4)+(2+6)+(2+8)$$

$$\sum_{t=1}^2 \sum_{i=1}^4 (t+2i) = 52$$

$$f) \sum_{i=1}^3 \frac{1+i}{i!} = \frac{2}{1} + \frac{3}{2} + \frac{4}{6} = \frac{2}{1} + \frac{3}{2} + \frac{2}{3} = \frac{12}{6} + \frac{9}{6} + \frac{4}{6} = \frac{25}{6}$$

Aufgabe 3:

$$a) \frac{3!}{5!} = \frac{1}{20} = 0,05$$

$$b) \frac{7!}{6!} = 7$$

$$c) \frac{(n+1)!}{n!} = n+1$$

$$d) \frac{n!}{n-1} = (n-2)! \cdot n$$

$$e) \frac{n!}{(n-1)!} = n$$

$$f) \frac{(n+1)!}{(n-1)!} = n \cdot (n+1)$$

$$g) (n+1) \cdot n! = (n+1)!$$

Aufgabe 4:

$$a) \binom{7}{4} = 35$$

$$b) \binom{30}{1} = 30$$

$$c) \binom{12}{10} = 66$$

$$d) \binom{12}{2} = 66$$

$$e) \binom{5}{3} + \binom{5}{4} = \binom{6}{4} = 15$$

$$f) \binom{3}{4} = n.d.$$

$$g) \binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

Aufgabe 5:

a) Beh.: $\binom{n}{0} = \binom{n}{n} = 1$

Beweis

(i) $\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{(n-0)! \cdot 0!} = \frac{n!}{n! \cdot (n-n)!} = \binom{n}{n}$

(ii) $\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \cdot n!} = 1$

b) Beh.: $\binom{n}{k} = \binom{n}{n-k}$

Beweis

$$\begin{aligned} \binom{n}{k} &= \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)! \cdot k!} \\ &= \frac{n!}{(n-k)! \cdot [n-(n-k)]!} = \binom{n}{n-k} \end{aligned}$$

c) Beh.: $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$

Beweis

$$\begin{aligned} \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} &= \frac{n! \cdot (k+1) + n! \cdot (n-k)}{(k+1)!(n-k)!} \\ &= \frac{n!(n+1)}{(k+1)!(n-k)!} = \frac{(n+1)!}{(k+1)!(n+k+1-1)!} \\ &= \frac{(n+1)!}{(k+1)! \cdot [n+1-(k-1)]!} = \binom{n+1}{k+1} \end{aligned}$$

Aufgabe 6:

$$\begin{aligned} a) \quad (x+2)^5 &= \binom{5}{0}x^5 + \binom{5}{1}2x^4 + \binom{5}{2}4x^3 + \binom{5}{3}8x^2 + \binom{5}{4}16x^1 + \binom{5}{5}32 \\ &= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32 \end{aligned}$$

$$\begin{aligned} b) \quad \left(\frac{1}{2}x - 4y\right)^4 &= \binom{4}{0}\frac{1}{16}x^4 + \binom{4}{1}\frac{1}{8}x^3(-4)y + \binom{4}{2}\frac{1}{4}x^2(16)y^2 + \binom{4}{3}\frac{1}{2}x^1(-64)y^3 + 256y^4 \\ &= \frac{1}{16}x^4 - 2x^3y + 24x^2y^2 - 128xy^3 + 256y^4 \end{aligned}$$