Partielle Integration

⇨ Mit ln-Funktion

$$\int u\,v'\,dx = u\,v - \int u'\,v\,dx \qquad (14)$$

A20

Beispiel 1:

$$I = \int x\,\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{1}{x}\frac{x^2}{2}\,dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + C,$$

$$u = \ln x,\quad u' = \frac{1}{x},\qquad v' = x,\quad v = \int x\,dx = \frac{x^2}{2}$$

Beispiel 2:

$$I = \int \ln^2 x dx = x\,\ln^2 x - 2\int \ln x\,dx = x\,\ln^2 x - 2x\,\ln x + 2x + C$$

$$\int \ln x\,dx = x\,(\ln x - 1) + C$$

$$u = \ln^2 x,\quad u' = \frac{2}{x}\ln x,\qquad v' = 1,\quad v = x$$

Beispiel 2:

$$I = \int \sqrt{x}\,\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx = \frac{2}{3}x^{3/2}\left(\ln x - \frac{2}{3}\right) + C =$$

$$= \frac{2}{3}x\,\sqrt{x}\left(\ln x - \frac{2}{3}\right) + C$$

$$u = \ln x,\quad u' = \frac{1}{x},\qquad v' = \sqrt{x},\quad v = \int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2}{3}x^{3/2}$$

a) $I_1 = \int \ln^3 x\,dx,\qquad I_2 = \int \ln^4 dx,\qquad I_3 = \int \ln^5 dx$

b) $I_1 = \int x^2 \ln x\,dx,\qquad I_2 = \int x^3 \ln x\,dx,\qquad I_3 = \int x\,\ln(x^2)\,dx$

c) $I_1 = \int (x+1)\ln x\,dx,\qquad I_2 = \int (x+1)^2 \ln x\,dx,\qquad I_3 = \int x\,\ln(x+1)$

d) $I_1 = \int \sqrt{x}\,\ln(\sqrt{x})\,dx,\qquad I_2 = \int \sqrt[3]{x}\,\ln x\,dx,\qquad I_3 = \int \sqrt[3]{x^2}\,\ln x\,dx$

A21

$$I_1 = \int \frac{\ln x}{x}\,dx,\qquad I_2 = \int \frac{\ln x}{x^2}\,dx,\qquad I_3 = \int \frac{\ln x}{x^3}\,dx$$

⇨ Mit Trigonometrischen Funktionen

A22

Beispiel 1:

$$I = \int x e^x dx = x e^x - \int e^x dx = (x-1) e^x + C,$$
$$u = x, \quad u' = 1, \quad v' = e^x, \quad v = \int e^x dx = e^x$$

$$I_1 = \int x^2 e^x \, dx, \qquad I_2 = \int x^3 e^x \, dx, \qquad I_3 = \int x^4 e^x \, dx$$

A23

a) $I_1 = \int x \sin x \, dx, \quad I_2 = \int x \cos x \, dx, \quad I_3 = \int x \sin(2x) \, dx, \quad I_4 = \int x \cos(2x) \, dx$

b) $I_1 = \int x^2 \sin x \, dx, \quad I_2 = \int x^2 \cos x \, dx, \quad I_3 = \int x^2 \cos(2x) \, dx, \quad I_4 = \int x^2 \sin(3x) \, dx$

Lösungen:

4.4. Partielle Integration

L20

a) $I_1 = \int \ln^3 x\,dx = x\ln^3 x - 3x\ln^2 x + 6x\ln x - 6x + C =$

$= x(\ln^3 x - 3\ln^2 x + 6\ln x - 6) + C$

$u = \ln^3 x,\quad u' = \frac{3}{x} \qquad v' = 1,\quad v = x$

$\int \ln^2 x\,dx = x(\ln^2 x - 2\ln x + 2) + C,$ aus dem Beispiel zur Aufgabe

$I_2 = \int \ln^4 x\,dx = x(\ln^4 x - 4\ln^3 x + 12\ln^2 x - 24\ln x + 24) + C,$

$u = \ln^4 x,\qquad v' = 1$

$I_3 = \int \ln^5 x\,dx = x(\ln^5 x - 5\ln^4 x + 20\ln^3 x - 60\ln^2 x + 120\ln x - 120) + C,$

$u = \ln^5 x,\qquad v' = 1$

b) $I_1 = \int x^2 \ln x\,dx = \frac{x^3}{9}(3\ln x - 1) + C, \qquad u = \ln x,\quad v' = x^2,$

$I_2 = \int x^3 \ln x\,dx = \frac{x^4}{16}(4\ln x - 1) + C, \qquad u = \ln x,\quad v' = x^3,$

$I_3 = \int x\ln(x^2)\,dx =$

c) $I_1 = \int (x+1)\ln x\,dx = \int \ln x\,dx + \int x\ln x\,dx = x\ln x\left(\frac{x}{2}+1\right) - \frac{x^2}{4} - x + C,$

$I_2 = \int (x+1)^2 \ln x\,dx = \int (x^2+2x+1)\ln x\,dx =$

$= \int x^2 \ln x\,dx + 2\int x\ln x\,dx + \int \ln x\,dx =$

$= x\ln x\left(\frac{x^2}{3}+x+1\right) - \frac{x^3}{9} - \frac{x^2}{2} - x + C,$

$I_3 = \int x\ln(x+1) = (x+1)\ln(x+1)\left(\frac{1}{2}(x+1) - 1\right) - \frac{x^2}{4} + \frac{x}{2} + \frac{3}{4} + C$

d) $I_1 = \int \sqrt{x}\,\ln(\sqrt{x})\,dx = \frac{1}{3}x^{3/2}\left(\ln x - \frac{2}{3}\right) + C = \frac{1}{3}x\sqrt{x}\left(\ln x - \frac{2}{3}\right) + C$

$I_2 = \int \sqrt[3]{x}\,\ln x\,dx = \frac{3}{4}x^{4/3}\left(\ln x - \frac{3}{4}\right) + C = \frac{3}{4}x\sqrt[3]{x}\left(\ln x - \frac{3}{4}\right) + C$

$I_3 = \int \sqrt[5]{x^2}\,\ln x\,dx = \frac{5}{7}x^{7/5}\left(\ln x - \frac{5}{7}\right) + C = \frac{5}{7}x\sqrt[5]{x^2}\left(\ln x - \frac{5}{7}\right) + C$

L21

$$I_1 = \int \frac{\ln x}{x}\,dx = \frac{1}{2}\ln^2 x + C,$$
$$I_2 = \int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}(\ln x + 1) + C,$$
$$I_3 = \int \frac{\ln x}{x^3}\,dx = -\frac{1}{4x^2}(2\ln x + 1) + C$$

L22

$$I_1 = \int x^2 e^x\,dx = e^x\left(x^2 - 2x + 2\right) + C,$$
$$I_2 = \int x^3 e^x\,dx = e^x\left(x^3 - 3x^2 + 6x - 6\right) + C,$$
$$I_3 = \int x^4 e^x\,dx = e^x\left(x^4 - 4x^3 + 12x^2 - 24x + 24\right) + C$$

L23

a)
$$I_1 = \int x\sin x\,dx = \sin x - x\cos x + C,$$
$$I_2 = \int x\cos x\,dx = \cos x + x\sin x + C,$$
$$I_3 = \int x\sin(2x)\,dx = \frac{1}{4}\sin(2x) - \frac{x}{2}\cos(2x) + C,$$
$$I_4 = \int x\cos(2x)\,dx = \frac{1}{4}\cos(2x) + \frac{x}{2}\sin(2x) + C$$

b)
$$I_1 = \int x^2\sin x\,dx = -x^2\cos x + 2\cos x + 2x\sin x + C,$$
$$I_2 = \int x^2\cos x\,dx = x^2\sin x - 2\sin x + 2x\cos x + C,$$
$$I_3 = \int x^2\cos(2x)\,dx = \frac{1}{2}\sin(2x)\left(x^2 - \frac{1}{2}\right) + \frac{x}{2}\cos(2x) + C,$$
$$I_4 = \int x^2\sin(3x)\,dx = \frac{1}{3}\cos(3x)\left(-x^2 + \frac{2}{9}\right) + \frac{2}{9}x\sin(3x) + C$$